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### Abstract

The coupling parameters of a system of high  $Q$  coupled cavities, such as a filter, can be determined by measuring the  $0^\circ$ ,  $\pm 90^\circ$ , and  $\pm 180^\circ$  phases of the input reflection coefficient. This paper describes a technique to measure these coupling parameters automatically by using an HP-9845B desk-top computer to control a frequency synthesizer, a network analyzer, and a frequency counter.

### Introduction

Accurate determination of couplings between electrical cavities is necessary in the design of generalized direct-coupled high  $Q$  cavity filters. This becomes more critical the higher the order of the filter and/or the higher the cavity mode employed. A quick and accurate method of measuring a set of coupling parameters within the filter's structure is essential to the correct final tuning of the filter. In 1975, Atia and Williams described how phase measurement of the input reflection coefficient could be employed to compute the coupling values of a set of coupled high  $Q$  cavities [1]. This paper extends this work by showing how a desk-top computer can be employed to control a frequency synthesizer, a network analyzer, and a frequency counter to automatically measure and compute the coupling parameters.

### Coupling Theory

A general lumped-element equivalent circuit of  $n$  coupled cavities is shown in Figure 1. All the cavities are synchronously tuned to the same resonant frequencies,  $\omega_0 = 1/\sqrt{LC}$ . By employing a narrowband approximation, the coupling matrix,  $M$ , is purely imaginary and frequency independent near  $\omega_0$ .

The input impedance of this network,  $Z_{11}^{(n)}$ , is given by

$$Z_{11}^{(n)} = \frac{\det(j\lambda I - jM_n)}{\det(j\lambda I - jM_{n-1})} \quad (1)$$

where  $M_{n-1}$  is the matrix resulting from the deletion of the first row and column of  $M_n$ ,  $\lambda = \omega - 1/\omega$ , and  $I$  is the identity matrix. Practically, the determination of the intercavity couplings  $M_{ij}$  can be accomplished by determining the poles and zeros of  $Z_{11}^{(n)}$ , which are measured via the reflection coefficient  $\rho^{(n)}$ . This follows because

$$\rho^{(n)} = [Z_{11}^{(n)} - R] / [Z_{11}^{(n)} + R] \quad (2)$$

and

$$\phi_\rho = \tan^{-1} (2RZ / (R^2 - Z^2)), \text{ where } Z_{11}^{(n)} = jZ$$

Therefore,  $\phi_\rho = 0$  at the zeros of  $Z_{11}^{(n)}$ , and  $\phi_\rho = 180^\circ$  at the poles of  $Z_{11}^{(n)}$ . Further, the parameter  $R$  (numerically equal to the input transformer constant squared) can be determined by measuring the frequencies where  $\phi_\rho = \pm 90^\circ$  because, at these values,

$$R = \pm Z \quad (3)$$

### Single-Cavity Case

For a single cavity, the reflection coefficient  $\rho^{(1)}$  as a function of frequency will follow the response in Figure 2(a), and the input impedance is given by

$$Z_{11}^{(1)} = j\lambda \text{ where } \lambda = \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)$$

The source impedance,  $R$ , can be determined by measuring the frequencies at the  $\pm 90^\circ$  frequency positions as  $\omega_{r2}$  and  $\omega_{r1}$ , and then substituting into equation (3) to give  $R = (\omega_{r2} - \omega_{r1})/\omega_0$ , because  $\lambda \approx 2[(\omega - \omega_0)/\omega_0]$ . This approximation is valid for narrow bandwidths.

### Two-Cavity Case

For two series-connected cavities

$$Z_{11}^{(2)} = j \left( \frac{\lambda^2 - M_{12}^2}{\lambda} \right)$$

and the response of the reflection coefficient  $\rho^{(2)}$  as a function of frequency is shown in Figure 2(b). The zeros,  $\lambda_z$ , are at  $\pm M_{12}$ , and the pole is at  $\lambda_p = 0$ . The  $\pm 180^\circ$  frequency positions are measured as  $\omega_{z2}$  and  $\omega_{z1}$ , and the  $0^\circ$  position is  $\omega_{p1} = \omega_0$ . Because

$$\lambda_{z1} - \lambda_{z2} = 2M_{12} = 2 \frac{(\omega_{z1} - \omega_{z2})}{\omega_0}$$

$M_{12}$  can be evaluated as

$$M_{12} = \frac{(\omega_{z1} - \omega_{z2})}{\omega_0}$$

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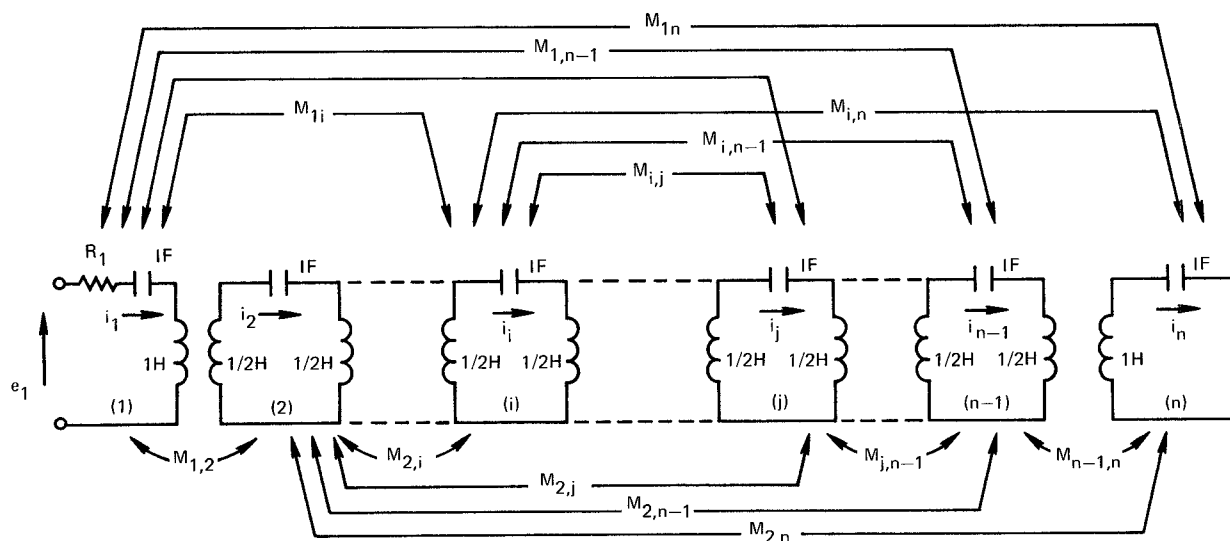


Figure 1. Equivalent Circuit of Coupled High Q Cavities

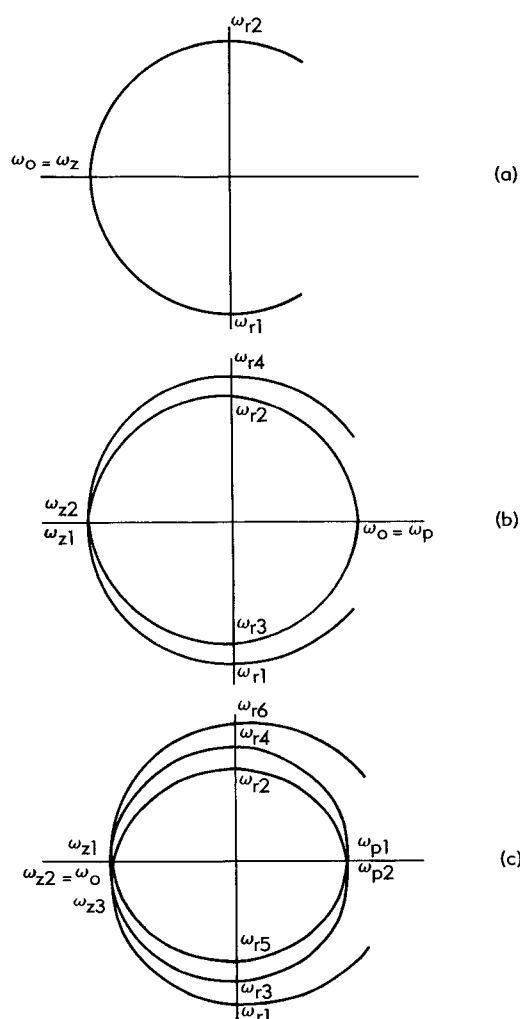


Figure 2. Reflection Coefficients as a Function of Frequency for One, Two, and Three Coupled Cavities

Therefore, the coupling  $M_{12}$  can be determined by measuring  $\omega_{z1}$ ,  $\omega_{z2}$ , and  $\omega_0$ . Further,  $R$  can be determined by evaluating  $Z_{11}^{(2)}$  at any of the  $\omega_{r1}$ ,  $\omega_{r2}$ ,  $\omega_{r3}$ , or  $\omega_{r4}$  frequency positions, since, at these points,  $R = \pm Z$ .

The procedure for one- and two-cavity cases can be extended to any number of series-connected cavities, and equations for the third, fourth, and fifth cases (see Figure 2(c) for the response of the third cavity) are given in Reference 1. Computations for lossy cavities to order 3 have also been made, and these are included in the program.

#### Measurement Procedure

The coupling parameters of a filter may be determined experimentally by employing a network analyzer, a stable source such as a frequency synthesizer, a frequency counter, and a desk-top computer such as an HP-9845B. A block diagram of a typical measurement setup is shown in Figure 3. A program has been written in BASIC for the Hewlett-Packard computer with the following logical sequence of steps:

**Step 1--**Load or input the coupling parameters of the filter to be measured. This includes entering the center frequency and bandwidth of the filter.

**Step 2--**Short circuit all the filter resonant cavities, and adjust the bandwidth of the sweep display to cover the frequency range of measurement.

**Step 3--**Enter the option to measure the first cavity, and select either a swept frequency display or a synthesizer display of the  $\pm 90^\circ$  theoretical frequency points.

**Step 4--**Tune the first resonant cavity for a symmetrical response, and let the computer automatically measure and compute the input  $R$ . Print out the result.

**Step 5--**Repeat the procedure and measure  $R$  and  $M_{12}$  by tuning the second resonant cavity and measuring the  $0^\circ$ ,  $\pm 90^\circ$ , and  $\pm 180^\circ$  frequency positions.

Step 6--Continue this procedure until all the coupling parameters have been measured. Note that each successive measurement represents a cross check on the preceding measurements.

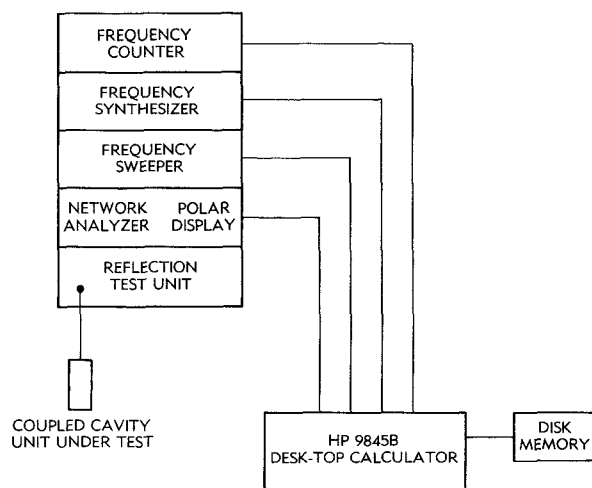


Figure 3. Block Diagram of Typical Measurement Setup

A flowchart of the computer program is given in Figure 4.

At each stage of the measurement procedure, the computer can be prompted to calculate the theoretical values of the  $0^\circ$ ,  $\pm 90^\circ$ , and  $\pm 180^\circ$  frequency positions for the particular reflection coefficient. These frequencies are shown on the polar display of the network analyzer. The short circuit network is tuned for symmetry and alignment of the theoretical frequencies closest to their correct location. The different locations of these values are a measure of the deviation of the filter coupling from the theoretical values.

#### Conclusions

This paper has presented a procedure whereby the coupling parameters of a high  $Q$  cavity filter can be automatically measured by using a computer-controlled microwave measurement circuit. This procedure offers a quick and accurate method of determining filter couplings. An additional bonus offered by this method is that it provides a major first step toward the final tuning of the filter.

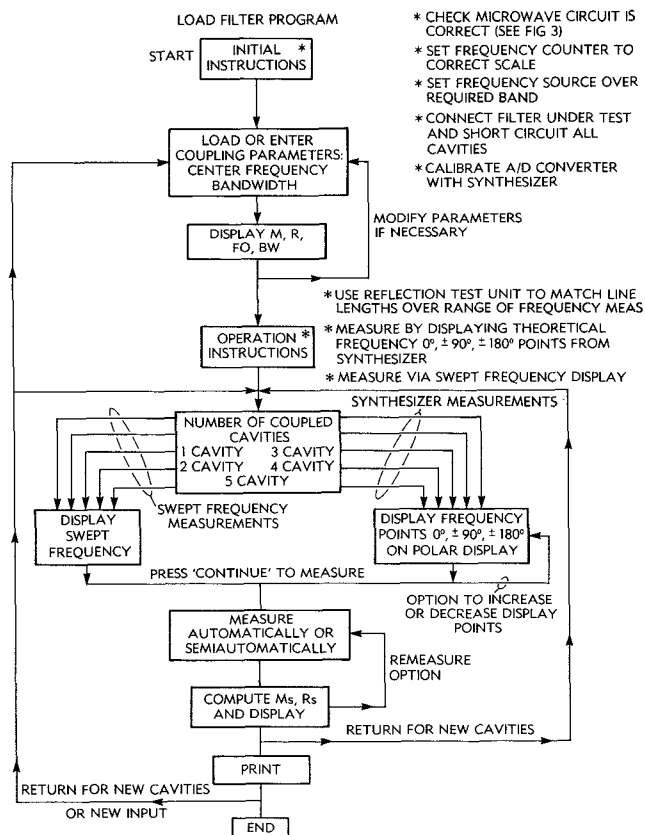


Figure 4. Simplified Flowchart of the Computer Program

#### Acknowledgments

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#### Reference

- [1] A. E. Atia and A. E. Williams, "Measurements of Intercavity Couplings," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-23, No. 6, June 1975.